Monte Carlo simulations of short-time critical dynamics with a conserved quantity

B. Zheng

FB Physik, Universität Halle, D-06099 Halle, Germany
and Institute of Theoretical Physics, Academia Sinica, 100080 Beijing, People's Republic of China

H. J. Luo

FB Physik, Universität Siegen, D-57068 Siegen, Germany

With Monte Carlo simulations, we investigate short-time critical dynamics of the three-dimensional antiferromagnetic Ising model with a globally conserved magnetization m_s (not the order parameter). From the power law behavior of the staggered magnetization (the order parameter), its second moment and the autocorrelation, we determine all static and dynamic critical exponents as well as the critical temperature. The universality class of $m_s = 0$ is the same as that without a conserved quantity, but the universality class of nonzero m_s is different.

I. INTRODUCTION

About ten years ago, in a pioneering work by Janssen, Schaub, and Schmittmann, short-time universal scaling behavior of nonequilibrium critical dynamics starting from a high-temperature state was systematically explored with renormalization group methods [1]. In a few years, extension of the calculations to different critical dynamics was carried out by Oerding and Janssen in a series of works [2–4]. Some evidences for the short-time dynamic scaling were also observed in Monte Carlo simulations [5–7]. Meanwhile, it was found that the power law decay of the magnetization starting from a completely ordered state has already emerged at relatively early times, e.g., see Ref. [8], and it can be used to estimate the dynamic exponent z [9,10]. In recent years short-time critical dynamics has been systematically investigated with Monte Carlo simulations [11–15]. Simulations have extended from regular classical spin models [14,16–19] to statistical systems with quenched disorder [20–22], quantum spin systems [23], lattice gauge theories [24], the hard-disk model [25,26], and dynamic systems without detailed balance [27–31]. The references given here are only a part of recent ones and not complete. A relatively complete list of the relevant references before 1998 can be found in a recent review, Ref. [14]. All numerical and analytical results confirm that there exists a rather general dynamic scaling form in critical dynamic systems already in the short-time regime of dynamic evolution.

Traditionally, it is believed that the short-time behavior of dynamic systems depends essentially on microscopic details. To understand short-time universal behavior, one should distinguish two different time scales, the macroscopic and microscopic time scales. The short-time dynamic scaling form emerges only after a time scale t_{mic}, which is sufficiently large in microscopic sense, but still very small in macroscopic sense. t_{mic} is a time that the system needs to sweep away the effect of microscopic details. In Monte Carlo simulations, for example, if a sweep over all spins on a lattice is considered to be a microscopic time unit, t_{mic} is usually the order of 10 to 100 Monte Carlo time steps [14]. For the simple Ising and Potts models with only nearest neighbor interactions, sometimes t_{mic} is negligibly small (e.g., one Monte Carlo time step or two). Compared with typical macroscopic time scales characterized by $\tau^{-\nu z}$ or L^z around the critical point, t_{mic} is indeed very small.

The physical origin of the short-time dynamic scaling is the divergent correlation time near a critical point. The divergent correlation time induces a memory effect, and the memory effect can be described by a scaling form. What does this scaling form look similar to? As pointed out by Janssen, Schaub, and Schmittmann [1], one should introduce new critical exponents to describe the dependence of the scaling behavior on the macroscopic initial conditions. Actually, for arbitrary initial conditions even a characteristic function is needed [32,33]. Furthermore, to describe the scaling behavior of some special dynamic observables, we also have to introduce new exponents, for example, the persistence exponent [34,35].

The short-time dynamic scaling form is not only conceptually interesting but also practically important. It provides new techniques for the measurements of both dynamic and static critical exponents as well as the critical temperature, for a review, see Ref. [14]. Since the measurements are carried out in the short-time regime of dynamic evolution, the dynamic approach does not suffer from critical slowing down.[1] Averaging is over both initial configurations and the random forces. This is very different from the time averaging in the measurements in equilibrium. Compared with those nonlocal methods which were invented to overcome the critical slowing down in Monte Carlo simulations in equilibrium, e.g., the cluster algorithms, the dynamic approach does study the original local dynamics and can be applied to disordered systems.

---

[1] In the literature, sometimes it is stated that the short-time dynamic approach can *eliminate* critical slowing down. Rigorously speaking, this statement is not correct. Critical slowing down always exists, at least for local dynamics, and it is the physical origin of the dynamic scaling. But in the short-time dynamic approach we do not have the problem of generating independent configurations. Therefore, we do not suffer from critical slowing down.

In fact, the dynamic scaling in nonequilibrium critical dynamics is not an exclusive phenomenon in the world. In many other nonequilibrium dynamic processes, universal or quasiuniversal scaling behavior has also been observed. An example is phase ordering dynamics [36]. In this case, a real equilibrium state and a macroscopic time scale such as $\tau^{-\nu z}$ does not exist. Therefore, ''short-time'' is not addressed. Another example is aging in complex systems such as glasses and spin glasses. Aging is just a kind of scaling or quasi-scaling. Dynamic scaling behavior around a spin glass transition [5,37–39,21] is very similar to that around a standard critical point. For example, the experimental measurements of the remanent magnetization in spin glasses support not only the power law scaling behavior but also the scaling relations between the exponents [40,21].

After such a slightly lengthy review, we come to what we are going to do in this paper. Even though renormalization group calculations have been extended to different critical dynamics, up to now, Monte Carlo simulations have been limited to dynamics of model A [41]. Actually, due to severe critical slowing down, Monte Carlo simulations for long-time behavior of critical dynamics beyond model A are also in the preliminary stage. The short-time dynamic approach provides powerful methods for numerical measurements of not only dynamic exponents, but also static exponents as well as the critical temperature. Simulations of dynamic systems beyond model A in equilibrium are usually difficult. Therefore, it is important to investigate short-time scaling behavior of these dynamic systems with Monte Carlo methods.

Dynamics of model A is a kind of relaxational dynamics *without* relevant conserved quantities [41]. If we consider only dynamic *relaxational* processes, dynamics with different relevant conserved quantities is classified into model B, C, and D. In this paper, taking the three-dimensional antiferromagnetic Ising model as an example with Monte Carlo methods, we study dynamics *with a conserved quantity*, which is *not* an order parameter but coupling to the order parameter. According to Ref. [41], this is called model C. In the next section, we introduce the model and analyze the power law scaling behavior in the short-time regime. In Sec. III, we present the numerical results. Section IV contains the concluding remarks.

II. SHORT-TIME DYNAMIC SCALING

A. The model

We consider an antiferromagnetic Ising model on a three-dimensional cubic lattice. The Hamiltonian of the model is

$$-H/kT = -K \sum_{\langle ij \rangle} S_i S_j, \qquad (1)$$

where S_i is an Ising spin and the sum is over nearest neighbors. For dynamics of model A and in equilibrium, on a cubic lattice the antiferromagnetic Ising model is equivalent to the (ferromagnetic) Ising model. Now, we keep the magnetization as a constant in dynamic evolution. Here the order parameter is *not* the magnetization but the staggered magne-

tization. We will denote the magnetization by m_s and the staggered magnetization by $M(t)$. For equilibrium states, it is expected that for $m_s = 0$, the universality class is the same as that of the standard Ising model without a conserved magnetization. For a nonzero m_s, according to Ref. [41], the critical exponent ν is different from that of $m_s = 0.0$, but it does not depend on the value of the nonzero m_s. For critical dynamics, the dynamic exponent z and the exponent θ for $m_s = 0$ are the same as those of the standard Ising model, but for nonzero m_s they are different. Again, z and θ do not depend on the value of the nonzero m_s. These conclusions are drawn from renormalization group calculations based on the ϕ^4 theory coupling to a conserved current [41,2]. It is interesting to compare these results with Monte Carlo simulations.

In this paper we study only dynamic relaxation starting from *disordered* states with a zero or small initial staggered magnetization m_0. We measure how the staggered magnetization and its second moment as well as the autocorrelation evolve in the dynamic process. The heat-bath algorithm is used in simulations. In order to keep m_s as a constant, in a flip we simply exchange the values of *two* spins. Here we should point out that if our model is not the antiferromagnetic but the ferromagnetic Ising model, the (conserved) magnetization m_s is the order parameter and then the dynamics for $m_s = 0$ belongs to model B [41].

In dynamics without any relevant conserved quantities, i.e., dynamics of model A, updating schemes are irrelevant in the sense of universality. Different updating schemes lead to the same critical exponents, either in an equilibrium or a nonequilibrium state. However, for dynamics with relevant conserved quantities, it is different, at least for nonequilibrium short-time behavior. In a recent work by one of the authors [42], for example, the Monte Carlo dynamics with a conserved order parameter for the two-dimensional Ising model is investigated. It is found that if in a flip we exchange the values of two *neighboring* spins, the dynamics is very slow. However, if we release the condition such that the order parameter is not locally conserved but only globally conserved (for example, in a flip we exchange the values of two randomly separated spins), the dynamic exponent is much smaller. For the dynamics studied in this paper, i.e., with a conserved quantity which is not the order parameter, the situation is somewhat similar. If in a flip we exchange the values of two neighboring spins, dynamic evolution is relatively slow. Therefore, as a first approach we decided to keep the magnetization m_s only globally conserved. In a flip we exchange the values of two randomly separated spins. Except for the updating schemes, all computational techniques adopted here are the same as those in the simulations of model A [14].

B. Short-time dynamic scaling

For critical dynamic systems, traditionally it is believed that universal scaling behavior exists only in the long-time regime of dynamic evolution. However, in recent years it is discovered that starting from *macroscopic* initial states, uni-

versal scaling behavior emerges already in the macroscopic short-time regime of dynamic processes after a microscopic time scale t_{mic} [1,5,6,11,12,14]. A typical example is that a magnetic system initially in a high-temperature state with a *small* initial order parameter m_0, is suddenly quenched to the critical temperature T_c or nearby (without external magnetic field) and then released to dynamic evolution of model A [41]. A generalized dynamic scaling form can be written down, for example, for the kth moment of the order parameter

$$M^{(k)}(t,\tau,L,m_0)=b^{-k\beta/\nu}M^{(k)}(b^{-z}t,b^{1/\nu}\tau,b^{-1}L,b^{x_0}m_0).$$
(2)

Here t is the time variable, τ is the reduced temperature, and L is the lattice size. β and ν are standard static exponents and z is the dynamic exponent. It is important that a new independent exponent x_0 is introduced to describe the scaling behavior of the initial order parameter m_0. If the scaling form above is valid, all relevant exponents can be extracted from the short-time behavior of relevant observables.

For the dynamic system with a conserved quantity discussed in the last subsection, we assume that a scaling form such as Eq. (2) also holds. In this case, $\tau=(K-K_c)/K_c$ and K_c depends on the conserved magnetization m_s, i.e., there is a critical line $K_c=K_c(m_s)$. In principle, there are two possibilities for the critical exponents. In the case of strong universality, the value of an exponent for nonzero m_s can be different from that for zero m_s, but does not depend on the value of the nonzero m_s. In the case of weak universality, an exponent may vary its value continuously along a critical line. For the ϕ^4 theory coupling to a conserved current in Ref. [41], it is the case of strong universality.

Neglecting the finite size effect and noting that m_0 is small, we expand the right hand side of Eq. (2) and take only the first nonzero (linear) term under the condition of a small t. At the initial stage of time evolution, the staggered magnetization around the critical temperature behaves as

$$M(t,\tau,m_0)\sim m_0 t^\theta F(t^{1/\nu z}\tau), \quad \theta=(x_0-\beta/\nu)/z. \quad (3)$$

At the exact critical point $\tau=0$, $M(t)$ obeys a power law $\sim t^\theta$. Numerical results and analytical calculations have revealed that the exponent θ is positive for almost all systems, i.e., the order parameter undergoes *an initial increase*. This makes the short-time behavior very prominent. The physical mechanism for this increase has not been very clear. At least the mean-field effect or symmetry breaking is not very relevant. Slightly away from the critical point, the power law behavior is modified by the scaling function $F(t^{1/\nu z}\tau)$. This fact allows us to locate the critical temperature and meanwhile to measure the critical exponent θ.

Differentiation of Eq. (3) leads to

$$\partial_\tau M(t,\tau,m_0)|_{\tau=0}=m_0 t^{c_d}\partial_{\tau'}F(\tau')|_{\tau'=0}, \quad c_d=1/\nu z+\theta. \quad (4)$$

From this power law behavior, one can determine the critical exponent $1/\nu z$.

Taking into account that the nonequilibrium spatial correlation length ($\sim t^{1/z}$) is small at the initial stage of time evolution, from Eq. (2) it can be derived that the second moment at T_c subjects to a finite size scaling

$$M^{(2)}(t,L)\sim L^{-d}t^y, \quad y=(d-2\beta/\nu)/z. \quad (5)$$

For simplicity, here we have set $m_0=0$. Another interesting dynamic observable is the autocorrelation

$$A(t)\equiv\frac{1}{L^d}\left\langle\sum_i S_i(0)S_i(t)\right\rangle. \quad (6)$$

At the critical temperature T_c, $A(t)$ decays by a power law [43]

$$A(t)\sim t^{-\lambda}, \quad \lambda=\frac{d}{z}-\theta. \quad (7)$$

It is interesting that even though we have set $m_0=0$, θ (i.e., x_0) still enters the autocorrelation. This is because x_0 is actually the scaling dimension of the *local* order parameter.

From Eqs. (5) and (7), we are able to estimate the static exponent β/ν and the dynamic exponent z. Then we complete the measurements of all the exponents and the critical temperature. For details of the above scaling analysis and a more systematic extension, readers are referred to Refs. [1,43] and the recent review article Ref. [14].

III. MONTE CARLO SIMULATIONS

A. $m_s=0$

For the antiferromagnetic Ising model, in equilibrium the averaged magnetization is zero. If we notice that the order parameter is the staggered magnetization, the thermodynamic fluctuation of the magnetization goes to zero in the thermodynamic limit (infinite volume). In nonequilibrium dynamic processes without any conserved quantities, if one starts from initial states with a zero magnetization m_s, the magnetization will remain zero. This is the case for usual macroscopic initial states, e.g., the high-temperature and low-temperature states. The fluctuation of the magnetization in the dynamic processes is also zero in thermodynamic limit. Therefore, one expects that the equilibrium state of the dynamic process with a conserved magnetization $m_s=0$ is in a same universality class of the model without a conserved quantity, and so is the dynamic universality class as well. The critical temperature is also not changed.

We perform simulations for $m_s=0$ in order to confirm the above expectation and to demonstrate our computational techniques. The heat-bath algorithm is adopted in simulations since it is faster than the Metropolis algorithm in the short-time regime. Lattice sizes are $L=64$ and 128. Within our maximum updating times, no finite size effects are observed. We remind the readers here that it is an advantage of the short-time dynamic approach that the finite size effects are easily controlled due to the small nonequilibrium spatial correlation length ($\sim t^{1/z}$).
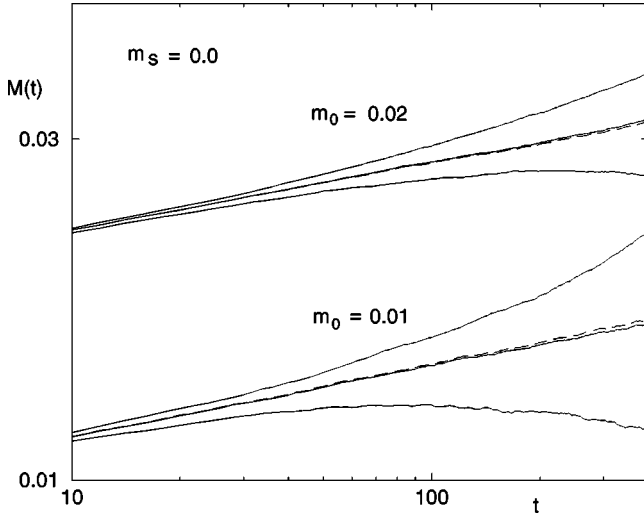
FIG. 1. Time evolution of the staggered magnetization displayed in log-log scale. For solid lines, K is 0.22065, 0.22165, and 0.22265 (from below) for $m_0=0.01$, and K is 0.22115, 0.22165, and 0.22215 for $m_0=0.02$. Dashed lines correspond to $K_c =0.22169(9)$ and 0.22163(5) for $m_0=0.01$ and $m_0=0.02$.
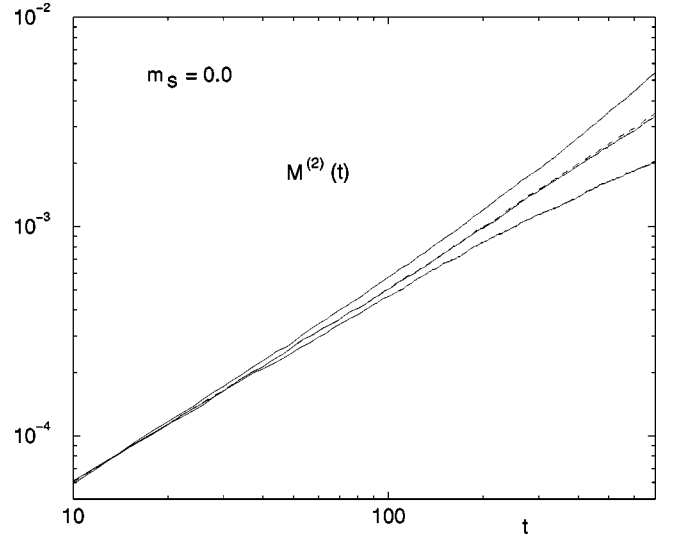

FIG. 2. The second moment in log-log scale. For solid lines, K is 0.22065, 0.22165, and 0.22265 (from below). The dashed line corresponds to $K_c=0.22164(22)$.

The critical temperature has been measured rather accurately in simulations in equilibrium (for the system without a conserved quantity). However, we also present data to determine it from short-time dynamics. With the short-time dynamic approach, in principle, any observables which are sensitive to the temperature around the critical regime can be used for the determination of the critical temperature.

In Fig. 1, time evolution of the staggered magnetization $M(t)$ is displayed with solid lines in log-log scale. The lattice size is $L=128$ and 2500 samples of initial configurations have been used for averaging. In order to see the possible "finite m_0 effects," we have performed the simulations with $m_0=0.01$ and 0.02. The coupling constant K is 0.22065, 0.22165, and 0.22265 (from below) for $m_0=0.01$, and 0.22115, 0.22165, and 0.22215 for $m_0=0.02$. In principle, the closer K's are used, the more accurate K_c and the critical exponent $1/\nu z$ can be obtained. However, we suffer from large statistical fluctuation if the K's are too close to each other. We use different K's for $m_0=0.01$ and $m_0=0.02$ just to study the possible systematic errors.

To make use of Eq. (3) to locate the critical point, we first interpolate $M(t)$ quadratically to any K around the three simulated ones, then search for a K which gives the best power law behavior for the curve. This K is K_c. The two dashed lines in Fig. 1 are the curves with the best power law behavior for $m_0=0.01$ and $m_0=0.02$. The corresponding K_c's are $K_c=0.22169(9)$ and 0.22163(5) for $m_0=0.01$ and $m_0=0.02$, respectively. These two values are consistent with each other and in agreement with that of the Ising model measured in equilibrium (without any conserved quantity), which is reported to be around 0.22165 [44,45]. We perform our measurements in a time interval $[t_{\mathrm{mic}},400]$. The results are rather stable when we take t_{mic} bigger than 10.

With K_c in hand, we measure the critical exponent θ $=0.108(5)$ and 0.100(5) for $m_0=0.01$ and $m_0=0.02$. They are consistent with $\theta=0.104(3)$ in Ref. [13] and θ

$=0.108(2)$ in Ref. [46] for the Ising model without a conserved quantity. Rigorously speaking, the exponent θ is defined in the limit of $m_0=0$. We should extrapolate θ to m_0 $=0$. However, since here there are some statistical errors, and especially the errors induced by the errors of the critical point K_c's, it is not meaningful to do so. To reduce the errors induced by K_c, we could take $K_c=0.22165$ as input. Then we obtain $\theta=0.106(6)$ by extrapolating the results to m_0 $=0$. In this paper, we simply consider $\theta=0.108(5)$ measured from $m_0=0.01$ as our final value of θ.

In Fig. 2, time evolution of the second moment of the staggered magnetization is displayed with solid lines in log-log scale. Here the initial staggered magnetization m_0 has been set to zero. From below, $K=0.22065$, 0.22165, and 0.22265. The lattice size is $L=64$ and the number of samples for averaging is 3200. Extra simulations for $L=128$ show that the finite size effect for $L=64$ is already negligibly small. From the figure, we see that the second moment is apparently less sensitive to K, compared with the staggered magnetization itself. However, we still can locate the critical K_c from the data. Measuring in a time interval $[t_{\mathrm{mic}}$ $=20,700]$, $K_c=0.22164(22)$. The curve corresponding to K_c is shown by a dashed line in the figure. The error of K_c here is bigger than that from the staggered magnetization. To reduce the error, simply increasing the samples for averaging is not enough, one must have longer maximum updating times and therefore larger lattices. These need much CPU times since the second moment is not self-averaging. To estimate the exponent $y=(d-2\beta/\nu)/z$, for simplicity, we take $K_c=0.22165$ as input. The result is $y=0.965(11)$.

In the simulations for Fig. 2, we have also measured the autocorrelation function $A(t)$. The results are shown in Fig. 3. From below, $K=0.22065$, 0.22165, and 0.22265. It is impossible to locate the critical point from the autocorrelation. We measure the exponent $\lambda=1.36(3)$ at $K_c=0.22165$ in a time interval $[20,100]$. When t is bigger than 100, the fluctuation becomes large.
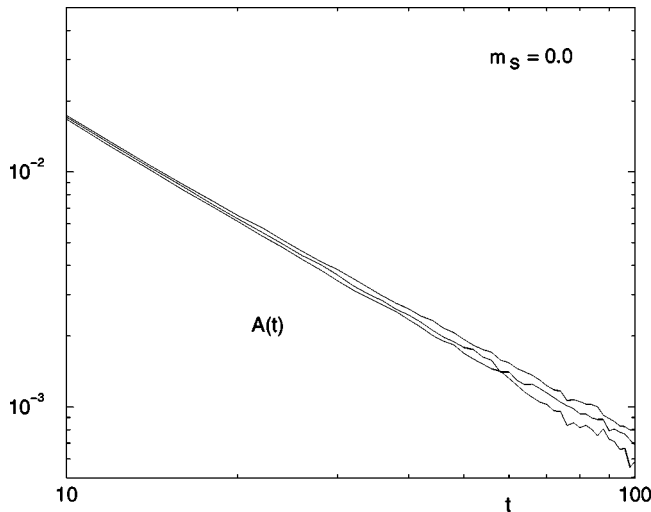
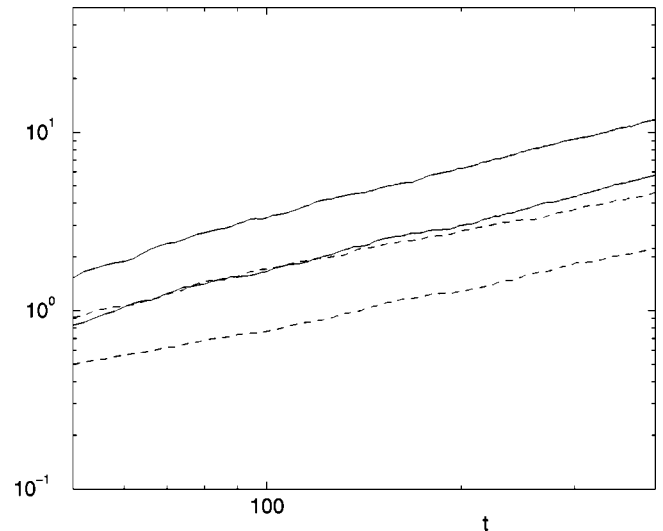FIG. 3. The autocorrelation in log-log scale. K is 0.22065, 0.22165, and 0.22265 (from below).

FIG. 4. The derivative $\partial_\tau M(t,\tau)|_{\tau=0}$ in log-log scale. Solid lines are for $m_s=0.0$ and from below, $m_0=0.01$ and 0.02. Dashed lines are for $m_s=0.2$ and 0.4 (from above) with $m_0=0.01$.

In order to determine the critical exponent ν, we need $\partial_\tau M(t,\tau)|_{\tau=0}$. From the data for Fig. 1, we can calculate this derivative approximately. The results are shown in Fig. 4 with solid lines in log-log scale. From below, the initial staggered magnetization is $m_0=0.01$ and 0.02. It is known that the power law behavior of the derivative of $M(t)$ in shorter times is less clean than that of $M(t)$ itself [14]. Therefore, we measure the slopes in a time interval [70,400] and obtain $c_d=1/\nu z+\theta=0.867(10)$ and 0.901(22) for $m_0=0.01$ and 0.02, respectively. The value from $m_0=0.01$ is more reliable.

In Table I, we list all the exponents we have measured. Taking the exponent θ as input, from λ we estimate the dynamic exponent z. Then from y and c_d we calculate the exponents $2\beta/\nu$ and ν. For comparison, available results for dynamics without a conserved quantity [46], i.e., the so-called model A dynamics, are also given, where the critical point K_c and the exponent ν are measured from another dynamic process starting from a completely ordered state. Within statistical errors, all the exponents for dynamics with and without a conserved quantity agree with each other well. The static exponents and K_c are also consistent with those of the standard Ising model measured from simulations in equilibrium with cluster algorithms [45]. Here we should make a comment. Even for dynamics of model A, the static exponents and K_c measured from the short-time dynamics are still not as accurate as those obtained with the cluster algorithms.

One reason is that the best effort for the short-time dynamic approach has not been made. Another reason is that in the short-time dynamic approach we have to measure both the static and dynamic exponents together. It makes the work more difficult than measuring only the static exponents. However, it is clear if we are interested in both static and dynamic properties, the short-time dynamic approach is very efficient. The cluster algorithms are nonlocal and change the dynamic universality class. On the other hand, the cluster algorithms cannot straightforwardly apply to any systems, e.g., the systems with quenched randomness and lattice gauge theories.

B. $m_s \neq 0$

Encouraged by the success for $m_s=0.0$, we proceed to $m_s \neq 0$. The results in the last subsection show that the lattices sizes $L=64$ and 128 are sufficient for our maximum updating times. The finite m_0 effect for $m_0=0.01$ is already invisible within the statistical errors. Therefore, in this subsection we will not systematically study the finite size and finite m_0 effects. However we have also performed some extra calculations to confirm the results presented. To locate the critical point K_c and estimate the exponent ν, we need data for several K's in the neighborhood of the critical point.

TABLE I. Critical exponents measured for different conserved magnetization m_s's. The values for dynamics of model A are taken from Ref. [46]. Under the item ''Ising'' are the values for the standard Ising model in equilibrium [45].

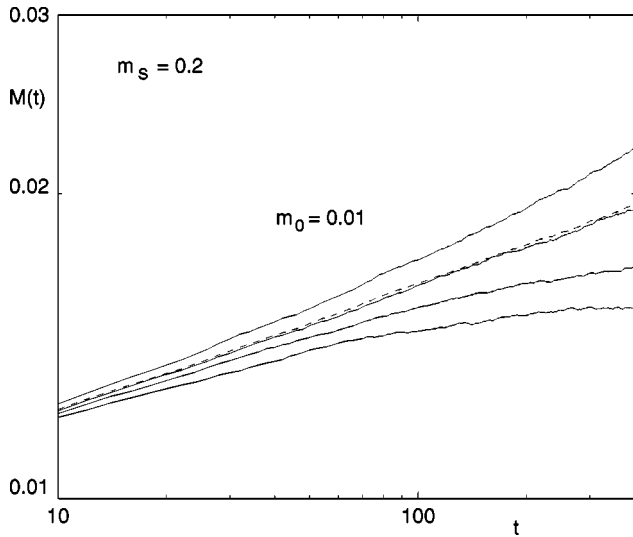| m_s	K_c	θ	λ	y	c_d	z	$2\beta/\nu$	ν
0.0	0.22169(9)	0.108(5)	1.36(3)	0.965(11)	0.867(10)	2.04(4)	1.03(4)	0.64(2)
0.2	0.24153(12)	0.129(5)	1.34(2)	0.954(4)	0.725(70)	2.04(3)	1.05(3)	0.82(8)
0.4	0.33230(20)	0.148(4)	1.31(1)	0.945(8)	0.748(20)	2.06(2)	1.05(3)	0.81(3)
Model A	0.22170(4)	0.108(2)	1.36(2)	0.970(11)		2.04(2)	1.034(4)	0.633(3)
Ising	0.221655(1)						1.037(3)	0.630(1)

FIG. 5. Time evolution of the staggered magnetization for m_s = 0.2 in log-log scale. For solid lines, K is 0.2405, 0.2410, 0.2415, and 0.2420 (from below). The dashed line corresponds to K_c = 0.24153(12).

The results in the last subsection also show that the difference of these K's should be around half a percent of K_c or less.

We perform simulations for m_s = 0.2 and 0.4. For m_s = 0.2 we obtain data for K = 0.2405, 0.2410, 0.2415, and 0.2420, which are shown in Fig. 5 with solid lines (from below) in log-log scale. For m_s = 0.4 we obtain data for K = 0.3310, 0.3320, 0.3330, and 0.3340, which are shown in Fig. 6 with solid lines (from below) in log-log scale. The lattice size is L = 128 and 2500 samples of the initial configurations are used for averaging.

Following the procedure in the last subsection, searching for a curve with the best power law behavior in a time inter-
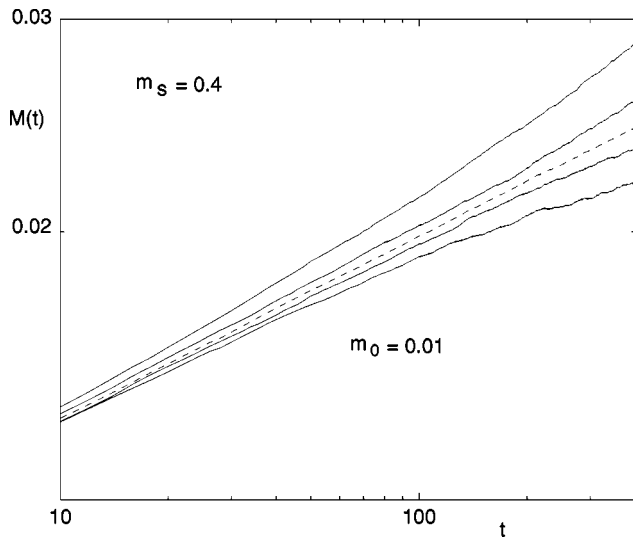

FIG. 6. Time evolution of the staggered magnetization for m_s = 0.4 in log-log scale. For solid lines, K is 0.3310, 0.3320, 0.333, and 0.3340 (from below). The dashed line corresponds to K_c = 0.33230(20).
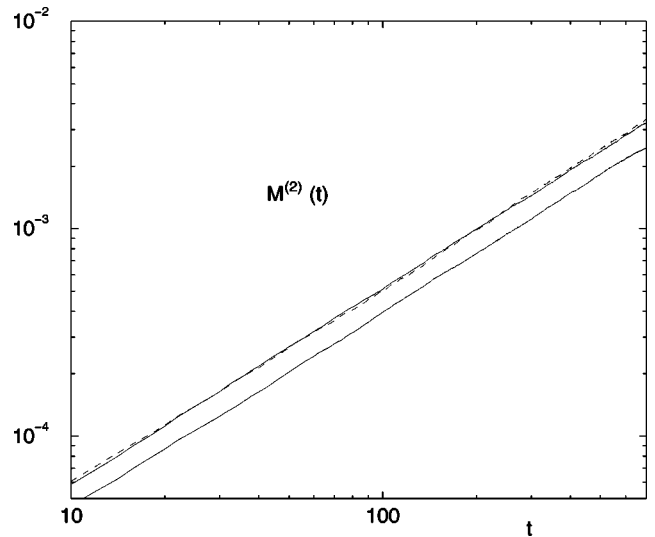

FIG. 7. The second moment at K_c in log-log scale. For solid lines, m_s = 0.2 and 0.4 (from above). The dashed line is for m_s = 0.0.

val $[t_{\text{mic}}, 400]$, one can determine K_c and the critical exponent θ. Now we have data for four K's and therefore have the choices to interpolate the staggered magnetization with three or four K's. For K_c, both yield the same values within errors. With four K's, we obtain K_c = 0.24153(12) for m_s = 0.2 and K_c = 0.33230(20) for m_s = 0.4 with t_{mic} = 20. The corresponding curves are shown in Figs. 5 and 6 with dashed lines. For m_s = 0.4, K_c is very stable for different choices of t_{mic}. For m_s = 0.2, K_c increases slightly when we take bigger t_{mic}. The reason is that our maximum updating times are not long enough. After carefully analyzing the data and looking at the curves in the figure, we are convinced that K_c = 0.24153(12) is the most reasonable.

In Fig. 4, $\partial_\tau M(t, \tau)|_{\tau=0}$ is displayed with dashed lines for m_s = 0.2 and 0.4 (from above). The results are obtained with four K's. For m_s = 0.4, the resulting exponent ν is bad with three K's. The reason is clear. K_c is between K = 0.3320 and 0.3330. If we choose only three K's, we have some systematic errors in interpolation.

With K_c in hand, we perform simulations for m_0 = 0.0 and measure the second moment and the autocorrelation. A lattice size L = 64 is used and the number of samples of initial configuration for averaging is 7500. In Fig. 7, the second moment is displayed in log-log scale with solid lines for m_s = 0.2 and m_s = 0.4 (from above). For comparison, the second moment for m_s = 0.0 is also plotted with a dashed line in the figure. Apparently, all three curves are parallel to each other. The exponent y is independent of m_s. In Fig. 8, the autocorrelation is displayed in log-log scale with solid lines for m_s = 0.2 and m_s = 0.4 (from below) and with a dashed line for m_s = 0.0. In all the relevant figures for nonzero m_s, good power law behavior is observed.

In Table I, all the exponents are summarized. Within the statistical errors, the static exponent $2\beta/\nu$ is independent of m_s. The values also coincide well with those obtained from dynamics of model A and measured in equilibrium for the standard Ising model. The static exponent ν for m_s = 0.2 and
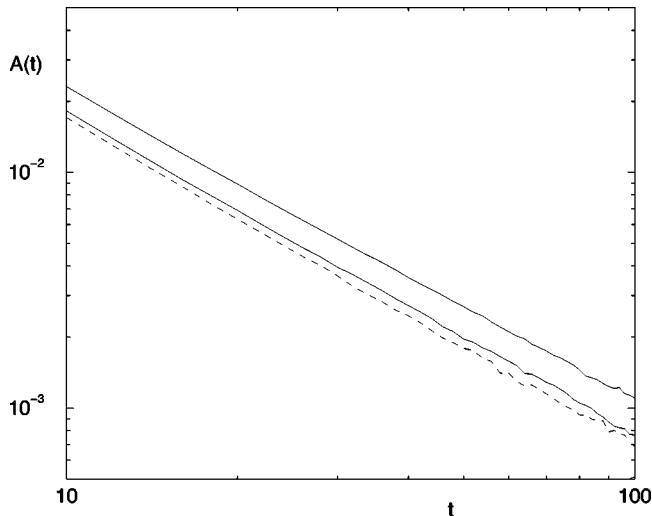
FIG. 8. The autocorrelation at K_c in log-log scale. For solid lines, $m_s = 0.2$ and 0.4 (from below). The dashed line is for $m_s = 0.0$.

0.4 are $0.82(8)$ and $0.81(3)$, respectively. Even though the first value carries a relatively big error, they are clearly different from $\nu = 0.64(2)$ for $m_s = 0.0$ and those measured from dynamics of model A and in equilibrium. However, ν is independent of the value of the nonzero m_s. These results are consistent with the theoretical calculations based on a ϕ^4 theory coupling to a conserved current [41]. With our accuracy, we cannot detect any dependence of the dynamic exponent z on m_s. This differs from the ϕ^4 theory in Ref. [41]. The exponent θ is $0.129(5)$ and $0.148(4)$ for $m_s = 0.2$ and 0.4, respectively. These two values are obviously different from $\theta = 0.108(5)$ for $m_s = 0.0$, and show nontrivial dynamic

behavior for nonzero m_s. However, with our data we should conclude that θ depends on the value of the nonzero m_s, even though there may be still some uncontrolled systematic errors. This point is different from the ϕ^4 theory.

IV. CONCLUDING REMARKS

We have reported our Monte Carlo simulations of short-time critical dynamics for the three-dimensional antiferromagnetic Ising model with a globally conserved magnetization (not an order parameter). The power law scaling behavior has been observed for the staggered magnetization (the order parameter), its second moment and the autocorrelation. All dynamic and static critical exponents are determined. For a conserved magnetization $m_s = 0.0$, all the exponents are the same as those of model A. For a nonzero m_s, the static exponents behave qualitatively the same as those of the ϕ^4 theory coupling to a conserved current [41]. However, the dynamic exponents are somewhat different. The reason might be that the magnetization is not locally but only globally conserved. Therefore, it is interesting to study the dynamics, with a locally conserved order parameter or non-order parameter: in a flip, we exchange two spins in a local regime, the size of which must be much smaller than the lattice size.

Note added in proof. P. Sen brought two relevant references to the attention of the author [47,48].

[1] H. K. Janssen, B. Schaub, and B. Schmittmann, Z. Phys. B: Condens. Matter **73**, 539 (1989).

[2] K. Oerding and H. K. Janssen, J. Phys. A **26**, 3369 (1993); **26**, 5295 (1993).

[3] K. Oerding and H. K. Janssen, J. Phys. A **27**, 715 (1994).

[4] K. Oerding and H. K. Janssen, J. Phys. A **28**, 4271 (1995).

[5] D. A. Huse, Phys. Rev. B **40**, 304 (1989).

[6] K. Humayun and A. J. Bray, J. Phys. A **24**, 1915 (1991).

[7] N. Menyhárd, J. Phys. A **27**, 663 (1994).

[8] D. Stauffer, Physica A **186**, 197 (1992).

[9] C. Münkel, D. W. Heermann, J. Adler, M. Gofman, and D. Stauffer, Physica A **193**, 540 (1993).

[10] N. Ito, Physica A **196**, 591 (1993).

[11] Z. B. Li, U. Ritschel, and B. Zheng, J. Phys. A **27**, L837 (1994).

[12] L. Schülke and B. Zheng, Phys. Lett. A **204**, 295 (1995).

[13] P. Grassberger, Physica A **214**, 547 (1995).

[14] B. Zheng, Int. J. Mod. Phys. B **12**, 1419 (1998), review article.

[15] B. Zheng, M. Schulz, and S. Trimper, Phys. Rev. Lett. **82**, 1891 (1999).

[16] J. B. Zhang, L. Wang, D. W. Gu, H. P. Ying, and D. R. Ji, Phys. Lett. A **262**, 226 (1999).

[17] B. Zheng, M. Schulz, and S. Trimper, Phys. Rev. E **59**, R1351 (1999).

[18] A. J. Bray, A. J. Briant, and D. K. Jervis, Phys. Rev. Lett. **84**, 1503 (2000).

[19] L. M. Jensen, B. J. Kim, and P. Minnhagen, Phys. Rev. B **61**, 15 412 (2000).

[20] H. J. Luo, L. Schülke, and B. Zheng, Phys. Rev. Lett. **81**, 180 (1998).

[21] H. J. Luo, L. Schülke, and B. Zheng, Mod. Phys. Lett. B **13**, 417 (1999).

[22] H. P. Ying and K. Harada, Phys. Rev. E **62**, 174 (2000).

[23] H. P. Ying, H. J. Luo, L. Schülke, and B. Zheng, Mod. Phys. Lett. B **12**, 1237 (1998).

[24] K. Okano, L. Schülke, and B. Zheng, Phys. Rev. D **57**, 1411 (1998).

[25] A. Jaster, Phys. Lett. A **258**, 59 (1999).

[26] A. Jaster, Physica A **277**, 106 (2000).

[27] N. Menyhárd and G. Ódor, J. Phys. A **29**, 7739 (1996).

[28] J. F. F. Mendes and M. A. Santos, Phys. Rev. E **57**, 108 (1998).

[29] T. Tomé and J. R. D. de Felício, Mod. Phys. Lett. B **12**, 873 (1998).

[30] T. Tomé and M. J. de Oliveira, Phys. Rev. E **58**, 4242 (1998).

[31] A. Brunstein and T. Tomé, Phys. Rev. E **60**, 3666 (1999).

[32] B. Zheng, Phys. Rev. Lett. **77**, 679 (1996).

[33] Y. Chen, S. H. Guo, Z. B. Li, and A. Ye, Eur. Phys. J. B **15**, 97 (2000).

[34] S. N. Majumdar, A. J. Bray, S. Cornell, and C. Sire, Phys. Rev. Lett. **77**, 3704 (1996).

[35] L. Schülke and B. Zheng, Phys. Lett. A **233**, 93 (1997).

[36] A. J. Bray, Adv. Phys. **43**, 357 (1994), and references therein.

[37] K. Fischer and J. Hertz, *Spin Glasses* (Cambridge University Press, Cambridge, England, 1991).

[38] R. E. Blundell, K. Humayun, and A. J. Bray, J. Phys. A **25**, L733 (1992).

[39] J. Kisker, L. Santen, M. Schreckenberg, and H. Rieger, Europhys. Lett. **20**, 159 (1992).

[40] P. Granberg, P. Svedlindh, P. Nordblad, L. Lundgren, and H. S. Chen, Phys. Rev. B **35**, 2075 (1987).

[41] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[42] B. Zheng, Phys. Lett. A **277**, 257 (2000).

[43] H. K. Janssen, in *From Phase Transition to Chaos*, edited by G. Györgyi, I. Kondor, L. Sasvári, and T. Tél, Topics in Modern Statistical Physics (World Scientific, Singapore, 1992).

[44] A. M. Ferrenberg and D. P. Landau, Phys. Rev. B **44**, 5081 (1991).

[45] H. W. J. Blöte, E. Luijten, and J. R. Heringa, J. Phys. A **28**, 6289 (1995).

[46] A. Jaster, J. Mainville, L. Schülke, and B. Zheng, J. Phys. A **32**, 1395 (1999).

[47] P. Sen, S. Dasgupta, and D. Stauffer, Eur. Phys. J. B **1**, 107 (1998).

[48] P. Sen, J. Phys. A **32**, 1623 (1999).